

MULTIYEAR POWER DISTRIBUTION PLANNING CONSIDERING VOLTAGE REGULATOR PLACEMENT

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Abstract

This paper proposes a multistage power distribution planning (PDP) method. The optimization method considers various planning alternatives that include the reinforcement of the existing feeders/substations, the construction of new distribution lines, and the placement of voltage regulators. The objective function aims at minimizing the net present value (NPV) of the network investment costs ensuring the safe operation of the distribution network throughout the planning period. The solution methodology consists of two phases. First, a mixed-integer quadratically constrained programming (MIQCP) model is formulated that determines the network investments, i.e., the reinforcement of substations and distribution lines, the network expansion plan and the voltage regulator (VR) placement at the final year of the planning period. Afterwards, a back-propagation method is adopted in order to define the year of commissioning the network investments that have been computed in the first phase. The efficiency of the proposed methodology is tested on a 22-bus distribution network under three different load growth scenarios.

Nomenclature

Sets

Ω_N	Set of buses.
Ω_{SS}	Set of substation buses.

Parameters

C^{SS}	Cost of substation reinforcement (€/MVA).
C^F	Capital cost of the distribution line (€/MVA·km).
C^{VR}	Investment cost of VR (€).
Inf	Inflation rate.
Int	Interest rate.
Le_{ij}	Length of the distribution line that connects buses i and j (km).
L_f	Loss factor of distribution line $i-j$.
M	Big number.

r_{ij} / x_{ij}	Resistance / reactance of distribution line $i-j$.
S_{ij}^{Fcap}	Existing capacity of distribution line $i-j$.
S_i^{SScap}	Initial capacity of the substation at bus i .
T	Duration of the planning period.
V_{min} / V_{max}	Minimum/maximum limit for voltage magnitude.

Continuous Variables

$P_{ij,t} / Q_{ij,t}$	Active/reactive power flow of line $i-j$ at period t .
$P_{i,t}^L / Q_{i,t}^L$	Active/reactive load of bus i at period t .
$P_{i,t}^{SS} / Q_{i,t}^{SS}$	Active/reactive power injected from the substation at bus i at period t .
$S_{ij,t}^{Ftot}$	Total capacity of line $i-j$ at period t .
$S_{i,t}^{SStot}$	Total capacity of the substation at bus i at period t .
$V_{sqr,i,t}$	The square of the voltage magnitude of bus i at period t .

Integer Variables

$S_{i,t}^{SS}$	Added capacity (MVA) to the substation at bus i at period t .
$S_{ij,t}^F$	Added capacity (MVA) to the line $i-j$ at period t .

Binary Variables

$y_{ij,t}$	Utilization variable that is equal to 1 if the power flows from bus i to bus j ; otherwise, it is equal to zero.
$z_{ij,t}^F$	Decision variable for reinforcement or addition of distribution line $i-j$ at period t .
$z_{i,t}^{SS}$	Decision variable for reinforcement of substation at bus i at period t .
$z_{ij,t}^{VR}$	Decision variable for placement of VR in distribution line $i-j$ at period t .

1 Introduction

The power distribution planning (PDP) problem consists of determining the optimal capacity, location and time period of new network investments in order to serve the future load demand in the most economic and reliable way. The optimal PDP is of high importance in order to avoid the overestimation of network investments, which can lead to costly planning solutions and idle network

assets. On the contrary, the underestimation of network investments can jeopardize the safe operation of the distribution network.

The distribution system is particularly important to the electrical utility company for two reasons: 1) its close proximity to the customer and 2) its high investment cost [1]. In the New Policies Scenario presented in [2], the investment on distribution amounts to 31% of the global investment in the power sector. The planning procedure seeks to minimize the investment cost and power losses subject to the operational constraints of the network. The PDP problem consists of a large majority of design variables, such as the location and capacity of the substations, distribution lines, voltage regulators (VRs) and capacitor banks (CBs) [3]. The integration of distributed generation (DG) units is also considered as planning alternative. The PDP is a complex problem, due to the number of technical feasible alternatives and the complicated configuration of large scale networks. Therefore, different planning methodologies based on mathematical optimization tools have been developed to find the optimum investment plan and minimize the cost of distribution system.

The PDP is a mixed integer nonlinear programming (MINLP) problem. An exhaustive review of PDP models and solution methodologies is presented in [3], [4]. There are two PDP models: 1) static and 2) multistage. In the static approach, PDP determines the planning requirements at the final year of the planning period. The multistage PDP model not only defines the capacity and location of the network investments, but also the appropriate year of commissioning them. The solution methodologies for the PDP problem can be divided into two categories: methods of mathematical programming (including non-linear, mixed integer and dynamic programming) and heuristic methods.

There are various approaches to the solution of the PDP problem depending on the models, the design variables, the number of the objectives and the solution methodologies. In [5], a mixed integer linear programming (MILP) model is used for the solution of the short term static PDP that considers the network expansion, reconductoring of distribution lines, and allocation of VRs and CBs. In [6], a simplified MILP multi-stage PDP model is presented that incorporates as planning options only the addition and reinforcement of substation and distribution lines in the presence of distributed generation (DG).

Some well-known heuristic methods that have been used for the PDP problem include: genetic algorithm [7], simulated annealing technique [8], and ant colony system algorithm [9]. A heuristic approach based on a back-propagation algorithm combined with a cost-benefit analysis is presented in [10] for the solution of the multistage PDP problem.

This paper proposes a long term multistage PDP optimization model that considers multiple planning alternatives. The proposed PDP is formulated as a mixed integer quadratically constrained programming (MIQCP) problem. The optimization procedure is divided into two phases. In the first phase, the

optimization model defines the optimal reinforcement of distribution lines and/or substations, network expansion and VR placement. In the second phase, the time period is determined for the commissioning of the network components derived from the first phase. The objective function aims at minimizing the net present value of the investment costs. The effectiveness of the proposed multistage PDP method is validated using a 22-bus distribution system and different load growth scenarios. The structure of the paper is as follows. In Section 2, the problem formulation is presented. In Section 3, the proposed solution methodology and the back-propagation algorithm are described. The numerical results of the method that was applied on a 22-bus distribution network are presented in Section 4. Conclusions are drawn in Section 5.

2 Problem Formulation

The following assumptions are considered for the proposed long term PDP problem:

- The loads of the distribution network are represented as constant real and reactive power.
- The flows of the active and reactive power on the distribution lines have the same direction.
- The distribution network is balanced.
- The set of possible connections for the future loads as well as the year that are connected to the network are given.

2.1 Objective Function

The objective function (1) of the problem represents the net present value (NPV) of the total investment costs, which correspond to the investment cost for the reinforcement of the substations (2), distribution lines (3) and voltage regulators (4):

$$\min J = \sum_{t=1}^T \left(\frac{1+Inf}{1+Int} \right)^t \cdot (F^{SS} + F^{Fdr} + F^{VR}) \quad (1)$$

$$F^{SS} = \sum_{i \in \Omega_{SS}} C^{SS} \cdot S_{i,t}^{SS} \quad (2)$$

$$F^{Fdr} = \sum_{i,j \in \Omega_N} C^F \cdot Le_{ij} \cdot S_{ij,t}^F \quad (3)$$

$$F^{VR} = \sum_{i,j \in \Omega_N} C^{VR} \cdot z_{ij,t}^{VR} \quad (4)$$

2.2 Problem Constraints

The proposed multistage PDP optimization model is subject to the following constraints:

$$\sum_{j \in \Omega_N} ((1-L_f) \cdot P_{ji,t} - P_{ij,t}) + P_{i,t}^{SS} = P_{i,t}^L \quad (5)$$

$$\sum_{j \in \Omega_N} ((1-L_f) \cdot Q_{ji,t} - Q_{ij,t}) + Q_{i,t}^{SS} = Q_{i,t}^L \quad (6)$$

$$V_{sqr,i,t} - V_{sqr,j,t} - 2 \cdot (r_{ij} \cdot P_{ij,t} + x_{ij} \cdot Q_{ij,t}) \leq M \cdot (1 - y_{ij,t}) \quad (7)$$

$$V_{sqr,i,t} - V_{sqr,j,t} - 2 \cdot (r_{ij} \cdot P_{ij,t} + x_{ij} \cdot Q_{ij,t}) \geq -M \cdot (1 - y_{ij,t}) \quad (8)$$

$$V_{\min}^2 \leq V_{sqr,i,t} \leq V_{\max}^2 \quad (9)$$

$$P_{ij,t} \leq M \cdot y_{ij,t} \quad (10)$$

$$P_{ji,t} \leq M \cdot (1 - y_{ij,t}) \quad (11)$$

$$Q_{ij,t} \leq M \cdot y_{ij,t} \quad (12)$$

$$Q_{ji,t} \leq M \cdot (1 - y_{ij,t}) \quad (13)$$

$$P_{ij,t} \cdot Q_{ij,t} \geq 0 \quad (14)$$

$$P_{ij,t}^2 + Q_{ij,t}^2 \leq (S_{ij,t}^{Ftot})^2 \quad (15)$$

$$S_{ij,t}^{Ftot} = S_{ij}^{Fcap} + \sum_{t'=1}^t (S_{ij,t'}^F + S_{ji,t'}^F) \quad (16)$$

$$S_{ij,t}^F \leq M \cdot z_{ij,t}^F \quad (17)$$

$$z_{ij,t}^F \leq y_{ij,t} \quad (18)$$

$$(P_{i,t}^{SS})^2 + (Q_{i,t}^{SS})^2 \leq (S_{i,t}^{SStot})^2 \quad (19)$$

$$S_{i,t}^{SStot} = S_i^{SScap} + \sum_{t'=1}^t S_{i,t'}^{SS} \quad (20)$$

$$S_{i,t}^{SS} \leq M \cdot z_{i,t}^{SS} \quad (21)$$

Equations (5) and (6) represent the active and reactive power balance at bus i at period t , respectively. The loss factor is used to approximate the power losses on a distribution line as a percentage of the incoming power. The square of the voltage magnitude of bus i at period t is calculated according to (7)–(8). Equations (5)–(8) are in fact the simplified *DistFlow* equations [11]. The constraints (10)–(14) ensure that the power flow on a line has only one direction. A disjunctive formulation is used in (7), (8) and (10)–(14) to avoid non-linear constraints in the problem. The thermal limits of the distribution lines are given by (15)–(18). The maximum apparent power that the substation can provide during the planning period is given by (19)–(21).

In this paper, it is assumed that each VR (Fig. 1) has a specific regulation range ($\pm rr\%$) with respect to the voltage reference magnitude. The VR losses are ignored and the VR tap change is considered as a continuous variable. The VR allocation is modeled as follows:

$$V_{sqr,i,t} \geq (1 - rr\%)^2 \cdot V_{sqr,m,t} \quad (22)$$

$$V_{sqr,i,t} \leq (1 + rr\%)^2 \cdot V_{sqr,m,t} \quad (23)$$

$$|V_{sqr,j,t} - V_{sqr,m,t}| \leq (V_{\max}^2 - V_{\min}^2) \cdot z_{ij,t}^{VR} \quad (24)$$

The VR's tap position is considered as continuous variable because the examined problem is a planning problem and not an operation one in order to consider the exact formulation of the VR. The objective of the proposed method is to find the location of the VRs and not to define thoroughly their operating status, since the uncertainties in the long term planning are very high.

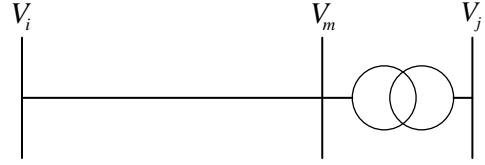


Fig. 1 Voltage regulator model

The radial configuration of the distribution network is ensured by (25)–(27), as follows:

$$\sum_{i,j \in \Omega_N} y_{ij,t} = |\Omega_N| - |\Omega_{SS}| \quad (25)$$

$$\sum_{j \in \Omega_N} y_{ji,t} = 1, \quad i \in \Omega_N \setminus \Omega_{SS} \quad (26)$$

$$y_{ij,t} = 0, \quad i \in \Omega_{SS} \quad (27)$$

3 Solution Methodology

The proposed multistage PDP methodology is divided into two phases. In the first phase, the optimal reinforcement of distribution lines and/or substations, network expansion and VR placement are determined for the maximum load of the final year of the planning period. The second phase determines the year of commissioning of the network investments that have been computed in the first phase, as shown in the flowchart of Fig. 2.

The steps of the proposed methodology are as follows:

- **Step 1:** A mixed integer quadratically constrained programming (MIQCP) model is formulated that minimizes the sum of (2) and (3) subject to the constraints (5), (6), (10)–(21), (25)–(27). The formulated problem is solved for the maximum load conditions of the final year. This step defines the location and the capacity of the distribution network assets that have to be reinforced and/or added in order to serve the load growth demand and the new loads.
- **Step 2:** A MILP model is formulated that minimizes (4) subject to the constraints (7)–(9), (22)–(24). The formulated problem is solved for the maximum load conditions of the final year. In this step, the placement of voltage regulators is examined using the network configuration that was derived from the first step.
- **Step 3:** In the final step, a heuristic method based on a back-propagation algorithm is performed in order to define the year of each element of set $\{H\}$ that contains the network investments that were calculated from the previous two steps. The planning procedure is shown in the flowchart of Fig. 2. This model has no integer or binary variables, since the investments and the network configuration were decided in the previous two steps.

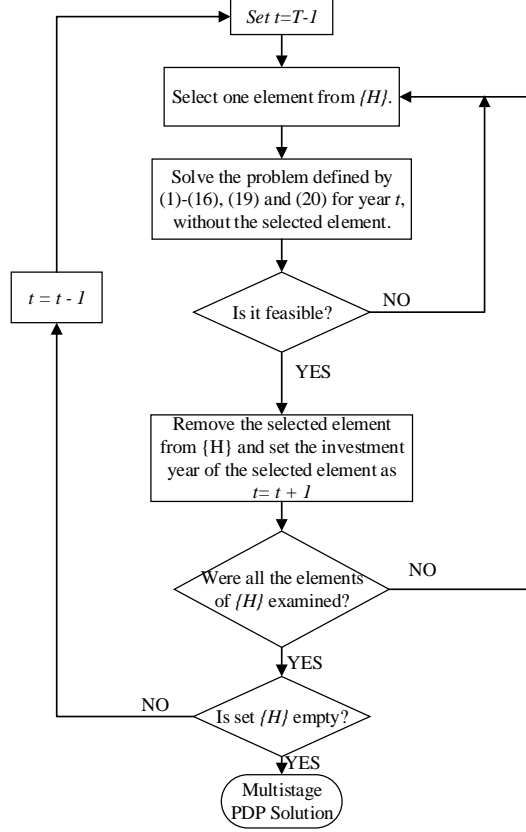


Fig. 2 Flowchart of the proposed method.

4 Numerical results

The proposed multistage PDP model has been developed in GAMS [12] using CPLEX solver. A 22-bus distribution network was used in order to validate its efficiency. The 20 kV network has one 25-MVA substation and 21 load buses and its topology can be seen in Fig. 3, where the square represents the substation, the continuous lines denote the initial network and the dashed lines denote the candidate distribution lines for the network expansion. The voltage limits are equal to $\pm 5\%$ of the nominal voltage. The maximum load at the reference year (year 0) is 5.37 MVA. The loss factor is considered to be equal to 2%. Table 1 presents the costs and technical characteristics of the available planning alternatives. The load demand of each bus at the reference year and the future loads as well as the year of their connection to the network are shown in Table 2. The data associated with the distribution lines of the network are presented in Table 3.

The method was tested for 3 different scenarios of load growth:

- Case 1: The annual load growth is considered 3% and 8 new loads are added.
- Case 2: The annual load growth is 3% at the first half of the planning period and 1.5% at the second half and 5 new loads are added.
- Case 3: Different annual load growth rates are

Table 1 Available conductors and VRs.

Conductors				
Type	R (Ω/km)	X (Ω/km)	Ampacity (A)	Cost ($\text{€}/\text{km}$)
1	1.268	0.422	136	10 000
2	0.576	0.393	224	15 000
3	0.215	0.334	445	23 000

Voltage regulators		
Type	Capacity (MVA)	Cost (€)
1	10	80 000

Table 2 Load data of the 22-bus distribution network.

Bus	P (MW)	Q (Mvar)	Year	Bus	P (MW)	Q (Mvar)	Year
1	0	0	0	16	0.15	0.07	0
2	0.50	0.24	0	17	0.15	0.07	0
3	0.25	0.12	0	18	0.07	0.03	0
4	0.18	0.09	0	19	0.15	0.07	0
5	0.25	0.12	0	20	0.25	0.12	0
6	0.29	0.14	0	21	0.11	0.05	0
7	0.14	0.07	0	22	0.11	0.05	0
8	0.36	0.17	0	23	0.14	0.07	5
9	0.11	0.05	0	24	0.14	0.07	7
10	0.58	0.28	0	25	0.06	0.03	5
11	0.15	0.07	0	26	0.10	0.05	8
12	0.15	0.73	0	27	0.10	0.05	2
13	0.25	0.12	0	28	0.06	0.03	8
14	0.15	0.07	0	29	0.10	0.05	2
15	0.18	0.09	0	30	0.10	0.05	3

Table 3 Branch data of the 22-bus distribution network.

From i	To j	Type	Le (km)	From i	To j	Type	Le (km)
1	2	3	1.5	22	23	-	1.4
2	3	3	1.6	22	24	-	1.4
3	4	3	1.0	23	24	-	0.7
4	5	3	2.0	9	25	-	1.5
5	6	3	1.6	10	25	-	1.5
6	7	3	1.1	18	25	-	1.8
7	8	3	0.9	13	26	-	1.8
8	9	3	1.7	21	26	-	1.5
9	10	1	1.0	23	26	-	1.1
10	11	1	0.8	8	27	-	0.8
11	12	1	0.5	18	27	-	1.1
12	13	1	0.8	25	27	-	1.1
13	14	1	0.7	18	28	-	1.0
14	15	1	1.0	19	28	-	1.0
15	16	1	0.6	10	29	-	1.8
16	17	1	0.5	11	29	-	1.5
6	18	1	2.3	26	29	-	0.5
18	19	1	1.0	24	30	-	0.5
19	20	1	1.0	29	30	-	1.5
13	21	1	1.4				
21	22	1	1.0				

considered for each bus and 3 new loads are added.

The planning period has duration of 20 years with an interest rate of 10% and inflation rate of 4%.

4.1 Case 1

Fig. 4 illustrates the obtained network topology by the end of the planning period. As shown in Table 4, the

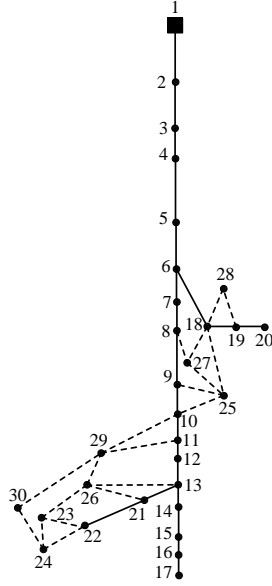


Fig. 3 The 22-bus distribution network

distribution line that connects bus 9 and bus 10 is reinforced with a type 2 conductor at year 12 and 8 distribution lines are added to connect the future loads. Furthermore, Table 4 shows that one VR is installed in the line that connects bus 9 and bus 10 at year 10 of the planning period. The NPV of the total investment costs is equal to 133 028 €, out of which the investment cost of the distribution lines is 85 676 € and the investment cost of the VR is 47 352. The total computational time was 32 s. Fig. 5 demonstrates the voltage profile along the feeder in the last year of the planning period with and without the installation of the VR. As shown in Fig. 5, the installation of the VR is necessary in order to keep the bus voltages between their limits.

4.2 Case 2

As shown in Table 5, there is no distribution line reinforcement in Case 2 and only the addition of 5 distribution lines is required in order to serve the future loads. Furthermore, one VR needs to be installed in the line that connects bus 12 and 13 at year 10 of the planning period. The total investment costs in Case 2 are equal to 94 872 \$ and the total computational time is equal to 23 s.

4.3 Case 3

In this case, different load growth rates are considered for each load bus. More specifically, the annual load growth rate of buses 2, 3, 5, 8–11, 13, 14, 17, 20–22, 24 and 25 is equal to 2%; the annual load growth rate of buses 4, 6, 7, 12 and 18 is equal to 3%; the annual load growth rate of buses 15, 16, 19 and 23 is 4%. As shown in Table 6, the line that connects buses 9 and 10 is reinforced with type 2 conductor at year 17 of the planning period and 3 new lines are added. The total investment costs are equal to 79 881 € and the computational time is equal to 15 s. Fig. 6 presents the

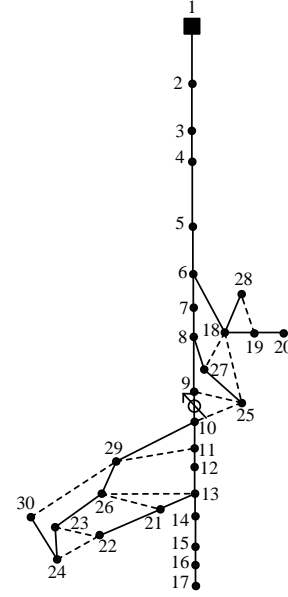


Fig. 4 The 22-bus distribution network topology by the end of the planning period in Case 1

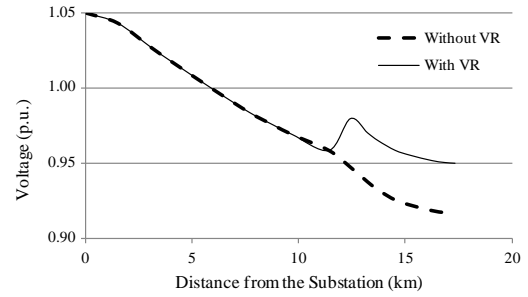


Fig. 5 Voltage profile of the 22-bus distribution network at the final year of the planning period in Case 1

Table 4 Results of the 22-bus distribution network in Case 1.

A. Distribution Lines			
From i	To j	Type	Year
9	10	2	12
26	23	1	3
23	24	1	3
27	25	1	5
29	26	1	3
8	27	1	2
18	28	1	8
10	29	1	2
24	30	1	3
B. VR			
From i	To j	Year	
9	10	10	

network configuration by the end of the planning period in Case 3.

5 Conclusion

Table 5 Results of the 22-bus distribution network in Case 2

A. Distribution Lines			
From i	To j	Type	Year
8	27	1	2
22	23	1	5
23	24	1	7
23	26	1	8
27	25	1	5

B. VR		
From i	To j	Year
12	13	10

Table 6 Results of the 22-bus distribution network in Case 3.

A. Distribution Lines			
From i	To j	Type	Year
9	10	2	17
22	24	1	5
23	24	1	5
10	25	1	5

B. VR		
From i	To j	Year
11	12	13

The proposed multistage PDP method simultaneously considers the following planning decisions: 1) reinforcement of substations/feeders, 2) construction of new distribution lines and 3) placement of voltage regulators. The method is divided into two phases. The first phase consists of two mixed integer programming (MIP) models and defines the capacity and the location of the planning investments for the maximum load condition. In the second phase, a heuristic approach with a back-propagation process is used to define the time of each investment.

Several cases with different load growth rates were examined. In the cases where a greater load growth was considered, the installation of VR proved to be necessary, thus increasing the total investment costs. The deviation in the total cost among the three cases indicates the significance of load forecast in the final investment plan.

With the use of the back-propagation algorithm the long term multistage PDP problem was solved in short computational time due to the absence of integer or binary variables, which is particularly important for large distribution systems.

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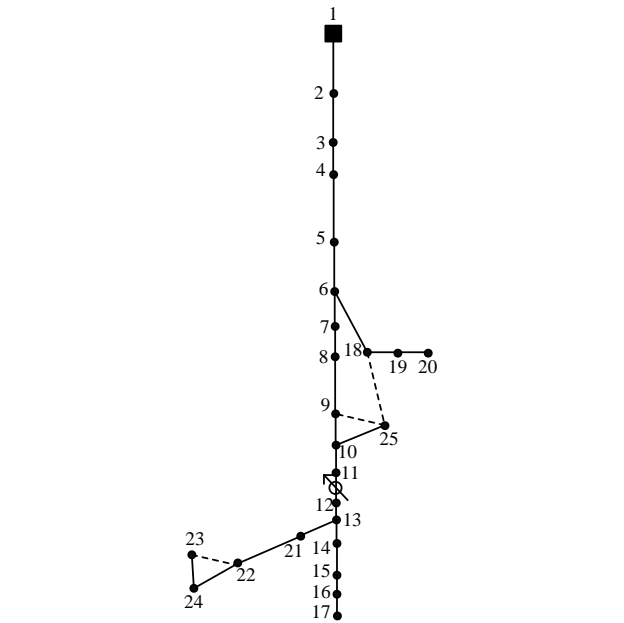


Fig. 6 The 22-bus distribution network topology by the end of the planning period in Case 3

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